Les droites telles que c a_1, c a_2, *font avec le plan tangent en* c *à* (E) *des angles dont le rapport des sinus est constant, quelle que soit la position de* c *sur* E.

Ce théorème étant vrai pour la ligne de courbure de (E), qui est dans l'un des plans principaux de cette surface, s'applique à des coniques homofocales ; par conséquent :

Etant données trois coniques homofocales, si d'un point c *de l'une, on mène une tangente à chacune des deux autres : le rapport des sinus des angles, que ces tangentes font avec la tangente en* c *à la première, est constant, quelle que soit la position de* c *sur cette courbe.*

Voici encore un moyen de faire voir comment sont liés entr'eux les centres de courbure principaux des trois surfaces homofocales à (O) qui passent par c.

Dans le plan ($a\ c\ b$) les points γ_2 et δ_2 sont les centres de courbure de deux coniques qui ont pour foyers les points a et b, et qui passent par c. On sait alors* que le point δ_2, relatif à l'une de ses courbes, est le pôle de la droite $c\ \gamma_2$ par rapport à l'autre courbe. Il résulte de là que la droite $\gamma_2\ \delta_2$ est perpendiculaire à la droite qu'on obtient en prenant la symétrique, par rapport à $c\ \gamma_2$, de la projection du diamètre $o\ c$ sur le plan ($a\ c\ b$).†

De même pour la droite $\gamma_1\ \delta_1$, elle est perpendiculaire au symétrique, par rapport à la normale $c\ l$, de la projection sur le plan $a_1\ c\ b_1$ du même diamètre $o\ c$. On peut alors dire :

Les droites $\gamma_1\ \delta_1$, $\gamma_2\ \delta_2$, *sont perpendiculaires à la direction suivant laquelle le diamètre* o c *serait réfléchi en* c *si la surface* (E) *était réfléchissante.*

II. "On Measuring the relative Thermal Intensity of the Sun, and on a Self-Registering Instrument for that Purpose." By E. FRANKLAND, D.C.L., F.R.S. Received January 24, 1882.

The thermometric estimation of relative solar intensity, according to the best known means, requires first the determination of the temperature of the air—so-called shade temperature—and secondly, and simultaneously, that of a thermometer with a blackened bulb placed *in vacuo* in the sunshine—sun temperature : the difference

* Voir "Treatise on Conic Sections." By Rev. G. Salmon. (6th edition, p. 56.)

† Pour arriver à ce résultat, il suffit d'appliquer au point δ_2 de la tangente $c\delta_2$, ce théorème dû à M. Ribaucour :—" D'un point m, pris arbitrairement sur la tangente en c à une conique, on abaisse des perpendiculaires sur la polaire de m et sur le diamètre aboutissant en c, elles interceptent, sur la normale en c, un segment égal au rayon de courbure en ce point."

between the two temperatures being taken as a measure of the sun's radiant heat operating at the time and place of the two observations.

The chief sources of error in this method are the difficulty of ascertaining the temperature of the air immediately surrounding the vacuous globe containing the blackened bulb, and the placing of this thermometer under exactly similar conditions at different meteorological stations. How considerable may be the errors arising from these sources will be evident from the following observations and experimental results.

Determination of Shade Temperature.

A thermometer merely shaded from the sun gives, in air of uniform temperature, readings differing very widely from each other according to its surroundings. If it be placed opposite a wall, for instance, upon which the sun is shining, the temperature indicated will be several degrees above what it would be if there were no such object near. I have also observed a difference in its readings when, on the one hand, it is exposed towards a blue sky, or, on the other, towards white clouds. Again, if the thermometer be placed in a louvred box, the readings will be much too high, unless the outside of the box be white; because the box becomes heated by the sun and communicates its heat to the air entering the louvres. Even the colour of the ground beneath the box has considerable influence upon the temperature of the air inside.

A true shade temperature means the temperature of free air in full sunshine; and, strictly, it ought to be ascertained without any shade at all, for, as soon as a shade is created, conditions supervene which often entirely baffle the object of the observer. The shade of a parasol exhibits a different temperature from the shade of a tree, and this again differs widely from that of a house. The temperature of the shade of a sheet of tinfoil is quite different from that of a sheet of writing-paper. Indeed, it may be truly said that every shade has its own peculiar temperature. The following thermometric readings show this effect of the area of shade, and of the quality of the shading material:—

Shade Temperatures.

Beneath larch tree	19·5° C.
,, white parasol	25·0
,, small white paper arch	35·0
,, small arch of bright tinfoil	45·2

Thus shade temperatures, measured during $1\frac{3}{4}$ hours of uninterrupted sunshine in the middle of the day, and within a few yards of the same spot, differed by no less than 25°·7 C. These observations

were, however, made at Pontresina, 5,915 feet above sea level, and so wide a range would probably not occur at lower altitudes.

The most effective shading material is, obviously, that which most perfectly reflects solar heat; and of all materials with which I have experimented white paper is the best, white linen and zinc-white being nearly equal to it. The most trustworthy shade thermometer, therefore, is one having its bulb covered with a thin layer of white paper, or, in default of this, the naked bulb may be shaded by a small arch of white paper. So placed, the thermometer will indicate a lower temperature than any obtainable in a similar shade produced by any other material.

The foregoing temperatures were observed when the thermometer was level with the ground, but the readings often rapidly become lower as the instrument is raised. The ratio of the diminution of temperature at increasing heights above the ground is, during sunshine, enormously great within a few feet of the earth. The ground, strongly heated by the sun, powerfully warms the molecules of air in immediate contact with it; these, becoming specifically lighter, rise, and at once begin to share their heat with the colder molecules above them, losing temperature in proportion as they mix with larger and larger volumes of supernatant cold air. The intensity of this effect attains a maximum when the air is calm, and a minimum during a storm. Indeed, these powerful convection currents are readily seen, on a calm sunny day, rising from the ground like the heated air from a stove, but they are scarcely, if at all, visible when a strong breeze is blowing.

In order to be comparable with each other, therefore, observations of shade temperature, whether at the same place or at different stations, should always be made under uniform conditions. That is to say, the thermometers, fully exposed to the air, should be similarly protected from radiant heat, and should be placed either at the level of the ground or at a definite height above it, upon a surface of uniform quality as regards absorbing and reflecting power. I would suggest that the bulb of the thermometer and 2 inches of its stem should be protected from the rain by being placed beneath a sheet-zinc arch of 1-inch span and 4 inches long, painted inside and out with "flatted" zinc-white. The instrument with the arch should be securely fixed horizontally upon a wooden stand 1 foot square, painted on both sides with "flatted" zinc-white, and, in order to avoid the excessively warm air very near the ground, the stand should have a height of 4 feet—an elevation convenient for observation, and one at which the temperature of the air suffers comparatively small decrements per foot of elevation. It should also be at a distance from buildings or trees, and have as free a horizon as possible. By the use of instruments so prepared and mounted, comparative and fairly

trustworthy determinations of air or shade temperatures in different localities would be obtained.

Determination of Sun Temperature.

The term sun temperature, as commonly employed, has a very vague meaning. If a body could be placed in sunlight under such circumstances as to absorb heat rays and emit none, its temperature would soon rise to that of the sun itself. But as all good absorbers of heat are also good radiators, the elevation of temperature caused by the exposure of even good absorbers to sunlight is comparatively small. Thus an isolated thermometer, with blackened glass bulb, placed in sunshine, will rarely rise more than 10° C. above the temperature which it marks when screened from direct sunlight. Under these circumstances, however, the thermometer loses heat not merely by radiation, but also by actual contact with the surrounding cold air. If the latter source of loss be obviated a much higher sun temperature is obtained; thus, the blackened bulb inclosed in a vacuous clear glass globe will sometimes, when placed in sunlight, rise as much as 60° C. above the shade temperature, and a still higher degree of heat may be obtained by exposing to the sun's rays the naked blackened bulb of a thermometer inclosed in a wooden box padded with black cloth, and closed by a lid of clear plate glass. Thus I obtained with such a box, on the 22nd of December, in Switzerland,* when the air was considerably below the freezing point, a temperature of 105° C., and a still higher temperature could doubtless be obtained by surrounding the thermometer with a vacuous globe before inclosing it in the padded box. These widely different temperatures, produced under different conditions by the solar rays, show that such observations can be comparative only when the thermometer employed to measure them is always surrounded by the same conditions. Under these equal conditions, however, the relative solar intensities at different times or places, are expressed by the number of degrees through which the sun's rays can raise the temperature of any body—the bulb of a thermometer, for instance—above that of the surrounding air. Various instruments have been contrived for such measurements, but the thermometer with blackened bulb *in vacuo* is the most convenient. As indications of solar intensity, however, its readings are, as I shall proceed to show, of little value if, as is sometimes the case, the instrument be simply placed upon grass, or if the shade temperature be not determined in immediate proximity to the vacuous bulb. The following experiments, made with a blackened bulb *in vacuo* verified at Kew Observatory, show how dependent upon the nature of the surface beneath it are the indications of this instrument. They were all made when the thermometer was placed

* "Proc. Roy. Soc.," vol. 22, p. 319.

horizontally with the stem at right angles to the direction of the sun's rays, and sufficient time was always allowed for the thermometer to assume its proper temperature before each reading was taken.

Tosten Vierod, near Laurwig, Norway.

July 17th. Brilliant sunshine, cloudless sky, strong breeze.

Time.	Position of thermometer.	Temperature.
9.40 A.M.	On green grass	57·3° C.
10.10 ,,	On somewhat parched grass	61·2
10.40 ,,	On bare soil	60·6
11.10 ,,	On staff 5 feet above meadow	50·5
11.40 ,,	On newly-mown grass	56·5
12.40 P.M.	On white paper	73·5
1.10 ,,	On staff as at 11.10	51·5

Wilhelmshöhe, Hesse Cassel.

Time 11 A.M. to 1 P.M. August 16th. Brilliant sunshine.

Position of thermometer.	Temperature.
On staff 5 feet above grass	51·8° C.
On black caoutchouc	54·7
On white paper	68·7
On glass mirror	64·0
On black silk	56·5
On slightly concave metallic mirror	64·0
On grass	58·5

Pontresina, Switzerland.

September 7th. Clear and cloudless sky; breeze.

Time.	Position of thermometer.	Temperature.
1.10 P.M.	On staff 4 feet above ice of Morta-ratsch Glacier	43·9° C.
1.30 ,,	On black caoutchouc laid upon glacier	39·0
2.0 ,,	On bare ice of glacier	47·5
2.30 ,,	On white paper laid upon glacier	53·0

Summit of Diavolezza Pass, Switzerland.

September 8th. Clear and cloudless except on horizon.

Time.	Position of thermometer.	Temperature.
10.30 A.M.	On staff 4 feet above snow	48·6° C.
11.0 ,,	On black caoutchouc laid upon snow	39·1
11.30 ,,	On bare snow	61·9
12.0 noon.	On white paper laid upon snow	65·5

Bellagio, Italy.

September 17th. Clear sky except near horizon. Air very moist and calm.

Time.	Position of thermometer.	Temperature.
10.30 A.M.	On black caoutchouc laid upon grass	60·0° C.
10.45 ,,	On black merino	59·0
11.15 ,,	On white paper	66·3
11.30 ,,	On white linen	66·0

In some observatories, the blackened bulb *in vacuo* is laid upon grass; but the experiments at Tosten Vierod show that its indications vary, under the same insolation, as much as 4°·7 C. according to the condition of the grass; somewhat parched, and consequently lighter coloured grass, giving a higher temperature than green and moderately long grass, whilst the latter raises the thermometer more than newly-mown grass. These differences result partly from differences of shade or air temperature in the immediate neighbourhood of the vacuous globe of the thermometer, and partly from the different reflecting power of the subjacent surface. With regard to the first of these causes, it must be borne in mind that the solar intensity is measured by the number of degrees through which the blackened bulb *in vacuo* is raised above the temperature of the medium immediately surrounding the vacuous globe. If this medium becomes warmer, the temperature of the blackened bulb will rise in a corresponding measure, and *vice versâ*, although the solar intensity remain the same. Now, the more absorbent the surface upon which the sun's rays fall, the higher, *cæteris paribus*, will be the temperature of the air resting upon that surface. Thus, with the same solar intensity, the shade or air temperature on white paper was 25°·2 C., on black caoutchouc 28°·5 C., on short grass 22°·7 C., and upon a rock 22°·6 C.; whilst at a height of 4 feet above the grass it was only 17°·9 C.

It is to the second of the causes just specified, however—the different reflecting power of the subjacent surface—that the variations of the sun thermometer under the same solar radiation are mainly due, as is proved by the following observations, in which the shade temperature was always taken under a small paper arch close to the vacuous globe:—

Suburb of Zürich. September 19th.

Time.	Position of thermometer.	Sun temperature.	Shade temperature.	Indicated solar intensity.
11.0 A.M.	Four feet above meadow.	47·3° C.	20·5° C.	26·8° C.
11.20 ,,	On grass	53·3	25·3	28·0
11.30 ,,	On white paper	67·0	25·3	41·7
12.0 noon	On black caoutchouc	59·0	27·0	32·0

These results show that the indications of the black bulb *in vacuo* are profoundly affected by the character of the surface beneath the instrument, the more perfect the reflecting power of that surface, other things equal, the higher the solar intensity indicated. Of all the substances tried, the highly reflecting power of white paper and linen for solar heat was very remarkable, exceeding appreciably that of bright metals, and even of freshly-fallen snow of dazzling whiteness. Of course, lateral reflection produces the same effect, and I found that the indicated solar intensity was increased by no less than 11° C. when the blackened bulb *in vacuo* was placed at a distance of 10 feet in front of a whitewashed wall upon which the sun was shining.

Finally the indications of the solar thermometer are also affected by strong wind, the readings of solar intensity being somewhat lower when the instrument is exposed to the current than when it is sheltered. The cause of this is obvious; the difference of temperature produced by the sun's radiant heat is *really* that between the inner blackened bulb and the glass of the vacuous globe. Now the latter is constantly receiving and absorbing obscure rays of heat from the blackened bulb, and its temperature must therefore always be somewhat higher than that of the surrounding air, which is measured by the shade-thermometer; but the glass globe will obviously maintain a less elevated temperature when it receives the strong impact of the molecules of cooler air in a breeze than when it is surrounded by a still atmosphere. The error thus introduced into the observations by a light breeze, however, does not seem to be serious, for I have not found it to exceed 0°·7 C.; but in a high wind it would probably be more considerable.

The results of the foregoing experiments disclose the precautions necessary to be observed to render such determinations of relative solar intensity fairly comparable and trustworthy. They are the following:—

1. The vacuous globe should always and everywhere be placed upon the same kind of horizontal reflecting surface.

2. The temperature of the air upon this reflecting surface should be taken as the shade temperature, and its observation should be synchronous with that of the sun-thermometer.

3. The horizon all round the instrument should be as free as possible, and there should be, especially, no sunlit walls in such a position as to reflect heat upon the thermometer.

4. As far as compatible with these conditions, the solar thermometer should be sheltered from the wind.

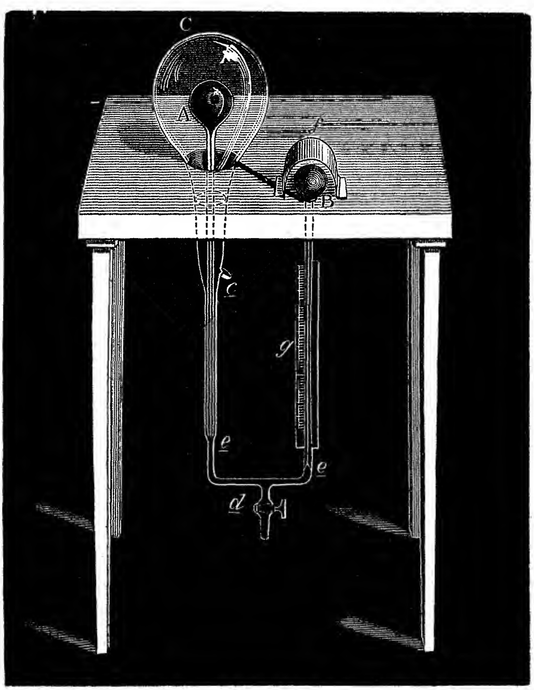
The white surface on which the thermometers are laid need not be of large area. A square foot practically affords to the blackened bulb *in vacuo* a reflective plane of infinite extent, for I have ascer-

tained that the indicated solar intensity is not augmented when the area of white surface is increased fourfold.

There is not much use in having *self-registering* shade and sun thermometers, because the highest temperature of the blackened bulb does not necessarily occur at the time of maximum shade temperature; and, consequently, the maximum solar intensity during any period cannot be found by merely deducting the maximum shade from the maximum sun temperature. Correct observations of maximum solar intensity are, therefore, very laborious with these instruments; and are, I believe, never made in the routine work of a meteorological station. As they afford, however, very interesting data, I have endeavoured to simplify them by contriving an instrument which allows them to be recorded for each day with one reading only.

A Differential Self-Registering Temperature, for measuring relative solar intensity.

This instrument, as seen from the accompanying figure, has considerable similarity to a Leslie's differential air thermometer; but in the new instrument the differential changes in the elasticity of the air of the two bulbs are measured by their action in elevating a column of mercury.



A, B are the two bulbs, 20 millims. in diameter, one of which, A, is blackened in the usual way, and then sealed into the larger clear glass globe, C, which has a small neck at *c* for attachment to a Sprengel pump. As soon as a good vacuum has been obtained, the neck is sealed off before the blowpipe, as shown in the figure. The other bulb, B, is shaded by a small zinc arch, *f*, 3 inches long, painted on both sides with "flatted" zinc-white. These bulbs are connected by a tube bent twice at right angles and furnished at *d* with a branch and stopcock.

The tube from the bulbs to *e e* is of the diameter of that of a self-registering spirit thermometer, but the remaining part of it is much wider, in order to diminish the friction of the column of mercury moving in it. The upright tube attached to B is provided with a scale, and the usual steel index is enclosed in the capillary part of it. The length of the scale is quite independent of the capacity of the bulbs, provided the volume of that part of the capillary tube in which the mercury oscillates is nearly a vanishing quantity in relation to the volume of the bulbs. It is well to have the two bulbs of nearly the same size, but a difference of capacity does not interfere with the accuracy of the instrument. The difference of the level of the mercury in the two limbs will be exactly proportional (neglecting the volume of the capillary tube) to the difference of temperature in the two bulbs, and the degrees of the scale, *g*, will therefore be equal throughout. The length of these degrees, however, though constant for any one instrument, will vary with the temperature and pressure at which the instrument has been filled, being greater the lower the temperature and the higher the pressure. At 0° C. and 760 millims. mercurial pressure, the difference of mercury level corresponding to 1° C. difference of temperature would be 2·784 millims., and the full length of these degrees may be practically obtained by making the wide portion of the tube longer in the limb A *e* than in B *e*, so that a minute depression of the mercury in A *e* will cause a great rise of the column in B *e*. The relative capacities of the capillary and wide tubes can be readily determined, and the necessary correction made in the readings for the depression of the mercury in A *e*.

The instrument may, however, be so constructed as to make the mercury rise and fall equally in both capillary tubes; in which case the rise in one limb would be accompanied by a corresponding fall in the other, and consequently the indicated degrees on the limb B *e* would be only half as long, 1·39 millims. corresponding to 1° C.—a graduation which is sufficiently open for all practical purposes. As a greater difference than 60° C. between the shade and sun thermometers has never been yet observed, a scale 163 millims. long in the one case, or half that length in the other, is sufficient, except perhaps for observations at very great altitudes.

As shown in the figure, the bulbs are supported upon a firm table,

one foot square and of any convenient height. A slot is cut in the table of suitable size to permit of the passage of the U-tube and stopcock, the aperture being subsequently closed by a slip of wood level with the top of the table. This table is painted on both sides with "flatted" zinc-white, and its legs are firmly fixed in the ground, so that it cannot be disturbed by the wind. The branch tube, *d*, should also be anchored to the ground by elastic bands, so as to prevent any movement of the instrument from the same cause. It is, moreover, desirable to surround the table with wire netting.

By means of a funnel and flexible tube, mercury is now introduced cautiously into the U-tube through the stopcock, *d*, until it reaches the zero of the scale, care being taken that both bulbs are at the same temperature (which should be noted) during the operation. The pressure upon the air in the bulbs must then be determined, and employed, together with the temperature, in calculating the length of each degree upon the scale. Once charged, the instrument must thenceforward be kept in its normal position, or nearly so, otherwise the mercury will get into the bulbs, whence it can only be dislodged with difficulty. In transporting the instrument from place to place it is therefore advisable to withdraw the mercury.

The following comparative determinations of solar intensity were made with this instrument, and with the ordinary blackened bulb *in vacuo* read on white paper synchronously with a shade thermometer.

Blackened bulb in vacuo.	Shade temperature.	Solar intensity.	
		Result of duplex observation.	Self-registration of differential instrument.
48°·3 C.	29°·0 C.	19°·3 C.	20°·0 C.
49·0	29·2	19·8	20·5
49·7	29·2	20·5	20·5
46·3	28·7	17·6	16·9
45·9	28·4	17·5	17·5
48·4	29·3	19·1	20·4
50·0	29·7	20·3	20·3
47·0	29·5	17·5	17·7
43·2	27·4	15·8	16·1